Rational Functions and Their Graphs
Section 3.5

Learning Objectives:

1. Find the domains of rational functions.
2. Use arrow notation.
3. Identify vertical asymptotes.
4. Identify horizontal asymptotes.
5. Use transformations to graph rational functions.
6. Graph rational functions.
7. Identify slant asymptotes.
8. Solve applied problems involving rational functions.

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1. Find the domain of the following rational function.

$$h(x) = \frac{x+9}{x^2-81}$$

2. Find the domain of the following rational function.

$$H(x) = \frac{16x^2+x}{x^2+5}$$

3. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

$$h(x) = \frac{x}{x+2}$$

4. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

$$h(x) = \frac{x+3}{x(x+9)}$$

5. Find the horizontal asymptote, if any, of the graph of the rational function.

$$f(x) = \frac{10x}{6x^2 + 5}$$

6. Find the horizontal asymptote, if any, of the graph of the rational function.

$$f(x) = \frac{21x^2}{7x^2 + 4}$$

7. Find the horizontal asymptote, if any, of the graph of the rational function.

$$f(x) = \frac{16x^3}{4x^2 + 3}$$

8. Graph the rational function. Answer part a. through b. below:

$$f(x) = \frac{6x}{x + 3}$$

a. What is the symmetry of this function?

_____

b. What is the y intercept, if any?

_____

c. What is/are the x-intercept(s), if any?

_____

d. What are the vertical asymptote(s), if any?

_____

e. What is the horizontal asymptote(s),if any?

_____

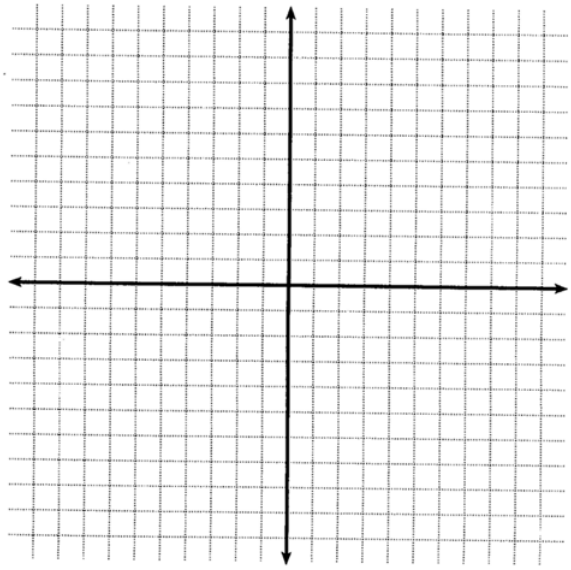
f. Find the y-coordinate for each of the following points:

$(-5, \_\_\_)$

$(-2, \_\_\_)$

$(5, \_\_\_)$

g. Using the information determined above, select the graph of the rational function.



9. Use the seven step method described in the book to graph the following rational function.

$$f(x) = \frac{7x}{x^2 - 16}$$

a. What is the symmetry of this function?

_____

b. What is the y intercept, if any?

_____

c. What is/are the x-intercept(s), if any?

_____

d. What is the equation of the vertical asymptote(s), if any?

_____

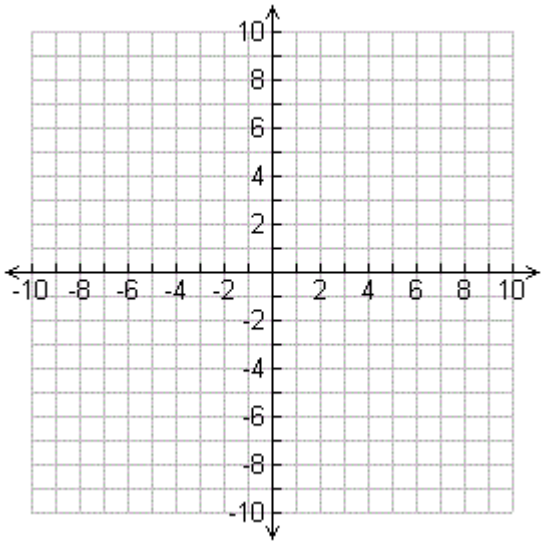
e. What is the equation of the horizontal asymptote(s) if any?

_____

f. Plot points between and beyond the x-intercept and the vertical asymptotes. Type the y-coordinate for each of the following points.

x	-5	-1	1/2	1	5
$f(x)$=					

g. Using the information determined above, select the graph of the rational function.



10. Use the seven step method described in the book to graph the following rational function.

$$f(x) = \frac{5x^2}{x^2 - 1}$$

a. What is the symmetry of this function?

_____

b. What is the y intercept, if any?

_____

c. What is/are the x-intercept(s), if any?

_____

d. What is the equation of the vertical asymptote(s), if any?

_____

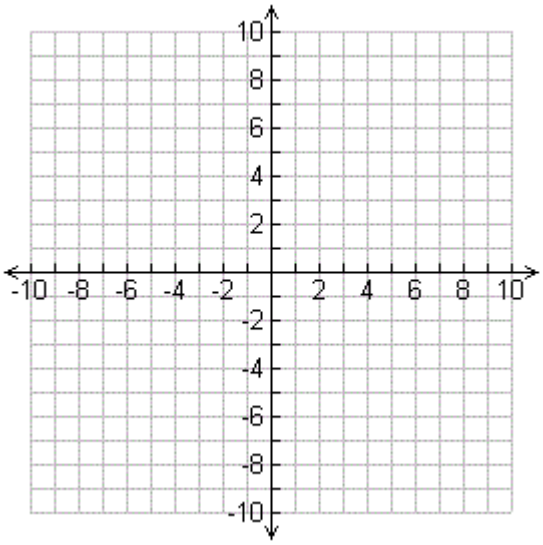
e. What is the equation of the horizontal asymptote(s) if any?

_____

f. Plot points between and beyond the x-intercept and the vertical asymptotes. Type the y-coordinate for each of the following points.

x	-4	-3	-1/2	3	4
$f(x)$=					

g. Using the information determined above, select the graph of the rational function.



11. Find the slant asymptote of the graph of the rational function and **b.** Use the slant asymptote to graph the rational function.

$$f(x) = \frac{x^2 + x - 12}{x - 7}$$

a. Find the slant asymptote of f, if any.

_____

b. Determine the symmetry of this function.

_____

c. What is the y intercept, if any?

_____

d. What is/are the x-intercept(s), if any?

_____

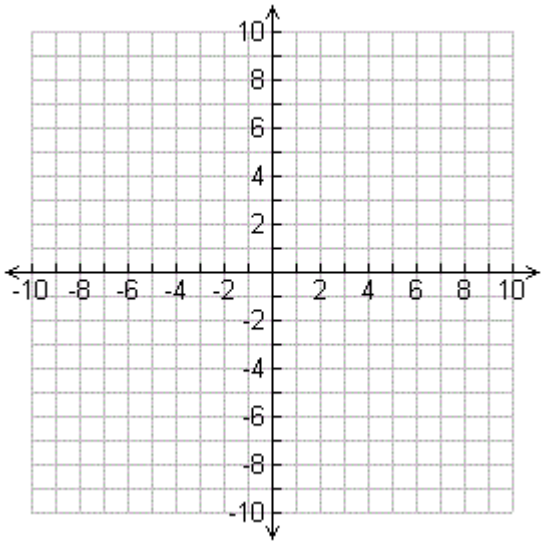
e. What is the equation of the vertical asymptote(s), if any?

_____

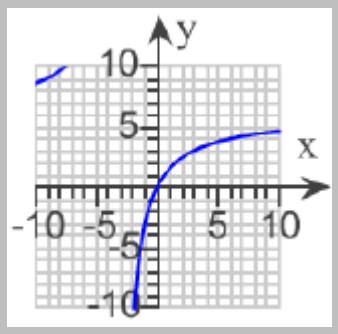
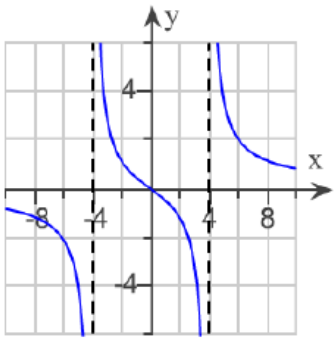
f. What is the equation of the horizontal asymptote(s) if any?

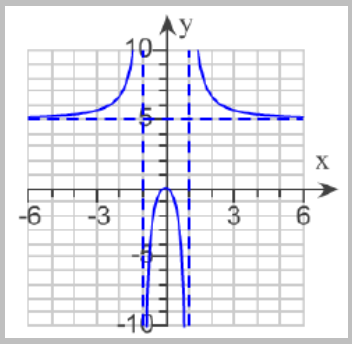
_____

g. Using the information determined above, select the graph of the rational function.



ANSWER KEY:

1.		The domain of h(x) is $\{x \mid x \neq 9, -9\}$.
2.		There is no restrictions on the domain.
3.		The vertical asymptote is $x = -2$. There are no holes.
4.		The vertical asymptote is $x = -9, x = 0$. There are no holes.
5.		The horizontal asymptote is $y = 0$.
6.		The horizontal asymptote is $y = 3$.
7.		There is no horizontal asymptote.
8.	a.	The function has no symmetry about the y-axis or the origin.
	b.	The y-intercept is 0.
	c.	The x-intercept is 0.
	d.	The x-coordinate of the vertical asymptote is -3.
	e.	The y-coordinate of the horizontal asymptote is 6.
	f.	$15, -12, \frac{15}{4}$
	g.	
9.	a.	The graph of f is symmetric with respect to the origin.
	b.	The y-intercept is 0.
	c.	The x-intercept is 0.
	d.	$x = -4, x = 4$
	e.	$y = 0$
	f.	$-\frac{35}{9}, \frac{7}{15}, -\frac{2}{9}, -\frac{7}{15}, \frac{35}{9}$
	g.	

10.	a.	The graph of f is symmetric with respect to the y-axis.
	b.	The y-intercept is 0.
	c.	The x-intercept is 0.
	d.	$x = 1, x = -1$
	e.	$y = 5$
	f.	$\frac{16}{3}, \frac{45}{8}, -\frac{5}{3}, \frac{45}{8}, \frac{16}{3}$
	g.	
11.	a.	$y = x + 8$
	b.	The graph has neither y-axis nor origin symmetry.
	c.	The y-intercept is $\frac{12}{7}$.
	d.	The x-intercepts are -4, 3.
	e.	$x = 7$
	f.	There is no horizontal asymptote.
	g.